# **Lesson Objectives**

* 1. The Basics of a Logarithm
  2. The Two “Special” Types of Logarithms
  3. Basic Properties of Logarithms
  4. Simplify (or evaluate) logarithms
  5. Convert Between Exponential and Logarithmic (and vice-versa)
  6. Solve Basic Logarithmic and Exponential Equations

# The **Basics** of a Logarithm

Suppose we have the exponential equation:

We know the answer is *x* = 3.

Because our variable is in the exponent, we don’t just use division to solve this equation:

NO, NO, NO!! Don’t do this!! → →

If you did, you would get *x* = 4. (incorrect) Division is used to undo multiplication, right?

But, we don’t have multiplication; we have exponential.

**BIG IDEA!** To undo an exponential function, we need to use its inverse – the **logarithm.**

A logarithm is an exponent.

Since a logarithm is an exponent, then it must necessarily have a **BASE.**

**Logarithmic form** (definition):

“**Logarithm** base *b* of ***x*** equals *y*”

Rewrite (or convert) to **exponential form**:

means the same thing as

**BIG IDEA!**  and are interchangeable in meaning.

# The Two **“Special”** Types of Logarithms

1. **Common** logarithm – base is **10**, but not explicitly written. It is understood to be 10.

If you see a logarithm written **without** a base, then the base is **10**.

* Examples: means means
* Calculator button is **LOG**  Picture of LOG button from Texas Instruments TI-83 Plus series or 84 Plus series graphing calculators. (to the left of the **7** button)
* This calculator button is **ONLY** for base **10**, the common logarithm!

1. **Natural** logarithm – base is *e*, but the logarithm is written as **“*ln*”** not “log*e*”.

* Examples: means means
* Calculator button is **LN** Picture of LN button from Texas Instruments TI-83 Plus series or 84 Plus series graphing calculators. (to the left of the **4** button)
* This calculator button is **ONLY** for base ***e***, the natural logarithm!

# Basic Properties of Logarithms

Recall **BIG IDEA!**: (these are interchangeable in meaning)

Here are some **Basic Logarithm Properties** to remember:

1. because (Any base with zero power is 1)
2. because (Any base to the power of 1 is the base itself)
3. because (Logarithm base *b* will undo exponential base b)

(Logarithm base b will undo “big” base b)

1. because (Exponential base b will undo log base b)

(“Big” base b will undo logarithm base b)

* **EXAMPLE:** Simplify the expression. [5.4-18]

means Property 1 Answer:

* **EXAMPLE:** Evaluate the logarithm. [5.4-15]

This is a natural logarithm (ln) – it has base *e*.

or means Property 1 Answer:

* **EXAMPLE:** Evaluate the logarithm. [5.4-14]

This is a natural logarithm (ln) – it has base *e*.

or means Property 2 Answer:

* **EXAMPLE:** Simplify the expression, if possible. [5.4.1]

Notice that the base of the logarithm is not written – it is a common logarithm, base 10.

(Logarithm base 10 will undo exponential base 10)

Property 3 Answer:

* **EXAMPLE:** Simplify the expression, if possible. [5.4.29]

This is a natural logarithm (ln) – it has base *e*.

(Logarithm base *e* will undo exponential base *e*)

Property 3 Answer:

* **EXAMPLE:** Find the indicated value of the logarithmic function.

[5.4.23]

(Logarithm base 7 will undo exponential base 7 )

Property 3 Answer:

* **EXAMPLE:** Simplify. [5.4.25]

(Exponential base 4 will undo logarithm base 4)

Property 4 Answer:

# Simplify (or Evaluate) Logarithms

* **EXAMPLE:** Find the logarithm [5.4.51]

Put “= *y*” on the end of the expression:

Chant: “A logarithm is an exponent.”

means: or

Since the value is a fraction, the exponent must be **negative**.

625 is a power of 5, since , or 54 = 625. So

# **Convert** between Exponential and Logarithmic (and vice-versa)

* **EXAMPLE:** Write in exponential form. [\*Lial 10.3.19]

Chant: “A logarithm is an exponent.”

What is the base? 10 What is the exponent? – 6 put them together:

What is the “value”? In exponential form:

* **EXAMPLE:** Write in exponential form. [\*Lial 10.3-11]

Chant: “A logarithm is an exponent.”

base = 15, exponent = 0, value = 1 In exponential form:

* **EXAMPLE:** Write in logarithmic form. [\*Lial 10.3-1]

base = 7, exponent = 3, value = 343

Chant: “A logarithm is an exponent.”

Setup:

In logarithmic form:

* **EXAMPLE:** Write in logarithmic form. [\*Lial 10.3-4]

base = 10, exponent = – 5, value = 0.00001

Chant: “A logarithm is an exponent.”

Setup:

In logarithmic form: or

# **Solve Basic Logarithmic and Exponential Equations**

## **Solve Basic Logarithmic Equations – ISOLATE and convert to EXPONENTIAL**

* **EXAMPLE:** Solve the equation. [5.4.95]

First, **ISOLATE** the logarithm. Divide by 9. (log has base 10)

Simplify. Do **NOT** divide by 3 in parentheses yet!

(It is stuck inside the logarithm.)

Chant: “A logarithm is an exponent.”

**Convert** the to exponential form with base 10.

Divide both sides by 3 and simplify.

Leave answer as a **fraction** – do not round. ANSWER:

* **EXAMPLE:** Solve the equation symbolically for the unknown. [5.4-30]

First, **ISOLATE** the logarithm. Divide by 3. (ln means base *e*)

Simplify. Do **NOT** divide by 4 yet!

(It is stuck inside the logarithm.)

Chant: “A logarithm is an exponent.”

**Convert** the to exponential form with base *e*.

Divide both sides by 4 and simplify.

Leave as exact answer with *e* – do not **round**. ANSWER: or

Answer is **NOT**: (the divide 4 is NOT part of the exponent!)

* **EXAMPLE:** Solve the equation. [5.4.105]

First, **ISOLATE** the logarithm. Subtract 4 both sides.

(do **NOT** do 4 – 2 = 2 at beginning!)

Combine like terms and simplify.

Divide both sides by – 2

Simplify.

Chant: “A logarithm is an exponent.”

**Convert** the to exponential form with base 3.

Simplify. ANSWER:

## **Solve Basic Exponential Equations – ISOLATE and convert to LOGARITHM**

* **EXAMPLE:** Solve the equation. Use the change of base formula as appropriate.

[5.4.73]

(Type an integer or decimal rounded to the nearest hundredth as needed.)

First, **ISOLATE** the exponential. Divide by 3.

Simplify. Do NOT round . Leave as **fraction** to the end!

Update the equation.

Do **NOT** divide by the 2 yet. It’s stuck in the exponential.

Chant: “A logarithm is an exponent.”

**Convert** the exponential to a logarithm base 10.

Divide both sides by 2 and simplify.

Simplify. Use calculator to round to hundredth.

This is a picture of the sequence of keys to press on the Texas Instruments TI-83 Plus or TI-84 Plus series calculators:
LOG 17 / 3 ) / 2 ENTER This is a screenshot from Texas Instruments TI-84 Plus CE calculator, but same result would occur with either TI-93 Plus or 84 Plus series calculators.
log(17/3)/2
answer returns:
0.3799938333

* **EXAMPLE:** Solve the equation. [5.4.81]

(Round to 4 decimal places as needed. Use a comma to separate answers as needed.)

**ISOLATE** the exponential. First, subtract 5.

Combine like terms and simplify.

Divide both sides by 5 and simplify.

Chant: “A logarithm is an exponent.”

**Convert** the exponential to a logarithm base *e*.

A logarithm base *e* is same as natural logarithm (ln)

Use calculator to get answer rounded to 4 decimal places. This is a screenshot from Texas Instruments TI-84 Plus CE calculator, but same result would occur with either TI-93 Plus or 84 Plus series calculators.
ln(3/5)
answer returns:
-0.5108256238

Answer:

* **EXAMPLE:** Solve the equation for *x*. [5.4-26]

(Type an integer or decimal rounded to the nearest hundredth as needed.)

Do **NOT** divide by – 1 yet. (It’s stuck in the exponential.)

No need to ISOLATE the exponential – it’s already there!

Chant: “A logarithm is an exponent.”

**Convert** the exponential to a logarithm base *e*.

A logarithm base *e* is same as natural logarithm (ln)

Divide by the – 1 both sides and simplify.

**INCORRECT:**

This is a screenshot from Texas Instruments TI-84 Plus CE calculator, but same result would occur with either TI-93 Plus or 84 Plus series calculators.
After entering
LN(-258)
a new window appears with error message caption for next picture.
If student selects "1:Quit" then screen will also read the word
"Error" This is a screenshot from Texas Instruments TI-84 Plus CE calculator, but same result would occur with either TI-93 Plus or 84 Plus series calculators.
After entering
LN(-258)
this screen appears with error message  that reads:
ERROR: NONREAL ANSWERS
1:Quit
2:Goto


That would be UNDEFINED.

In general, you ***cannot*** take logarithm of zero or negative. Only works!

is “exact” answer

Use calculator to round This is a screenshot from Texas Instruments TI-84 Plus CE calculator, but same result would occur with either TI-93 Plus or 84 Plus series calculators.
-ln(258)
answer returns:
-5.552959585

Answer:

Sources Used:

1. MyLab Math for *Algebra for College Students*, 8th Edition, Lial, Pearson Education Inc.
2. MyLab Math for *College Algebra with Modeling and Visualization*, 6th Edition, Rockswold, Pearson Education Inc.